tour is both simple and numerically efficient. For less regular boundaries, a good practice would be to locate more sources in the vicinity of the irregularities in the contour's shape and have these sources closer to the boundary.

Number of Sources and Matching Points: Let us denote by N_s the number of sources in the SES for a certain region. Also, let N_e and N_h be, respectively, the number of matching points for the electric and magnetic fields on the region's boundary. Having $N_s = N_e = N_h$ for each SES is quite convenient and useful in most cases, but is not mandatory. The only restriction is that the total number of matching points (notice that each matching point is common to two adjacent regions) be equal to or larger than the total number of sources so as to have a sufficient number of equations. Despite the freedom afforded by the above, the size, shape, and composition of the region should be taken into consideration when setting N_s, N_e, and N_h. For metallic regions, since only the electric field has to be matched (to zero) at the boundary, let $N_s = 0$ and $N_h = 0$. The number N_e is a function of the curvature and the length of the boundary. If the radius of curvature is smaller than $\lambda/5$, one should fix the matching points approximately half the radius of curvature apart. On the other hand, if the radius of curvature is larger than $\lambda/5$, one should consider about ten matching points per wavelength. For dielectric regions, the number of matching points $N_e = N_h$ is also a function of the boundary curvature and length. Again, if the radius of curvature is smaller than $\lambda/5$, one should fix the matching points approximately half the radius of curvature apart. On the other hand, if the radius of curvature is larger than $\lambda/5$, one should consider about ten matching points per wavelength in the region. Thus, a region of a higher permittivity would usually require more matching points. The number of sources can be equal to the number of matching points.

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TE–TM Mode Conversion of an Optical Beam Wave in Thin-Film Optical Waveguides

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***Abstract* —This paper describes the TE–TM mode conversion efficiency when a Gaussian beam wave propagates in thin-film optical waveguides. For film thicknesses at which strong coupling between the TE and TM modes is obtained, two hybrid modes have oppositely rotating circular polarizations, or linear polarizations perpendicular to each other with equal magnitude of TE and TM wave components. In the former (i.e., circular polarization), complete TE–TM mode conversion is impossible. In the latter (i.e., linear polarization) complete TE–TM mode conversion is available. These claims are based on the fact that the direction of power flow of the hybrid modes depends on the polarization.**

Manuscript received September 22, 1987; revised November 23, 1987.
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IEEE Log Number 8719442.

I. Introduction

Many studies have been made on thin-film optical waveguide devices utilizing the principle of TE–TM mode conversion, such as optical switches, modulators, and circulators [1]–[9]. For efficient TE–TM mode conversion, phase matching between the TE and TM modes is required. For this purpose, anisotropic media are used in part (or all) of the waveguide.

In general, the eigenmodes of the waveguide containing anisotropic media are hybrid modes. The phenomenon of the TE–TM mode conversion can be described as coupling between two hybrid modes [5], [6]. But, the direction of power flow of the hybrid modes is generally different from that of the wave-normal (a phenomenon called "walk off") [7]–[10]. Moreover, the direction of power flow of one of the two hybrid modes is not always the same as that of the other mode [11]. Therefore, when an optical beam wave is employed in the TE–TM mode conversion, it is expected that the coupling between the two hybrid modes gradually ceases as they travel down the waveguide even if both wave-normals are in the same direction. But, the conversion efficiency taking into account these effects has not been reported so far.

In this short paper we investigate TE–TM mode conversion efficiency when a Gaussian beam wave propagates in a thin-film optical waveguide with uniaxial anisotropic substrates. The diffraction of the beam wave is neglected. The uniaxial anisotropic substrates are gyrotropic or anisotropic media, and the dielectric tensor has longitudinal or polar configuration [1]. As the analysis of the hybrid modes in the thin-film waveguides has been reported [2], [5]–[10], only the conversion efficiency is given. It will be shown that complete TE–TM mode conversion is impossible for circularly polarized hybrid modes and that complete TE–TM mode conversion is available for linearly polarized hybrid modes.

II. TE–TM Mode Conversion

Fig. 1 shows a thin-film optical waveguide consisting of three sections: I (input), II (converter), and III (output). The top layer and film are lossless isotropic dielectric media with refractive indices of n_t and n_f, respectively. The substrate is a lossless uniaxial anisotropic dielectric medium having ordinary and extraordinary refractive indices of n_o and n_e, respectively. The permeabilities of the three media are equal to μ_0 (permeability of vacuum). The optic axis of the uniaxial anisotropic medium is parallel to the y axis for $z < 0$ and $l < z$, but is at an angle ζ with respect to the y axis for $0 < z < l$, as shown in Fig. 1. Then, the eigenmodes are TE and TM modes in sections I and III, and are hybrid modes in section II. The film thickness of the waveguide is taken to $d = d_0$, at which TE_0 and TM_0 modes are degenerate in sections I and III and two hybrid modes H_0^+ and H_0^- can propagate in section II. At the film thickness d_0, the two H_0^+ and H_0^- modes have right- and left-handed circular polarizations for longitudinal gyrotropic and polar anisotropic cases, or linear polarizations perpendicular to each other with equal magnitude of the TE and TM wave components for longitudinal anisotropic and polar gyrotropic cases [2]. Here, the direction of power flow

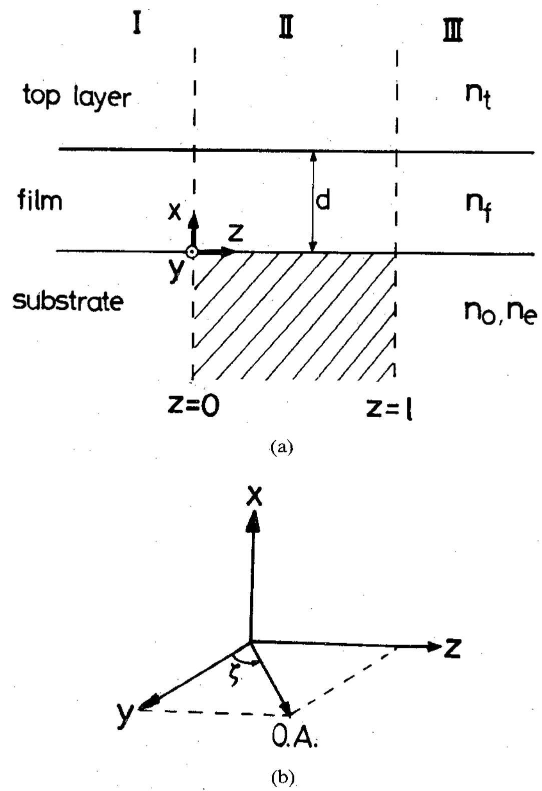

Fig. 1. (a) Thin-film optical waveguide consisting of three sections: I (input), II (converter), and III (output). In the substrate, $\zeta = 1°$ for $0 < z < l$ and $\zeta = 0°$ for $z < 0$ and $l < z$. (b) The orientation of the optic axis of uniaxial anisotropic substrate.

TABLE I

TRANSVERSE COMPONENT OF POWER FLOW (P_y) AND POLARIZATION STATES OF HYBRID MODES FOR REAL OR IMAGINARY OFF-DIAGONAL ELEMENT OF UNIAXIAL ANISOTROPIC DIELECTRIC TENSOR IN POLAR OR LONGITUDINAL CONFIGURATION

Off-diagonal element (ϵ_{12} or ϵ_{23})	Polar (ϵ_{23})	Longitudinal (ϵ_{12})
real (anisotropic)	$P_y \neq 0$ circular polarization	$P_y = 0$ linear polarization
imaginary (gyrotropic)	$P_y = 0$ linear polarization	$P_y \neq 0$ circular polarization

of the hybrid modes depends on the polarization [11]. The transverse component of the power flow (P_y) and the polarization states of the hybrid modes for the off-diagonal elements of uniaxial anisotropic dielectric tensor are listed in Table I.

The electric field of TE_0 mode with Gaussian distribution in the y direction is expressed as [12], [13]

$$E_y = e_y(x) G(y) \exp[j(\omega t - \beta z)] \quad (1)$$

where $e_y(x)$ is the electric field distribution function in the x direction [14], and

$$G(y) = -\frac{1}{\sqrt{\sqrt{\pi} W}} \exp\left[-\left(-\frac{y}{\sqrt{2} W}\right)^2\right] \quad (W\text{: beam width}). \quad (2)$$

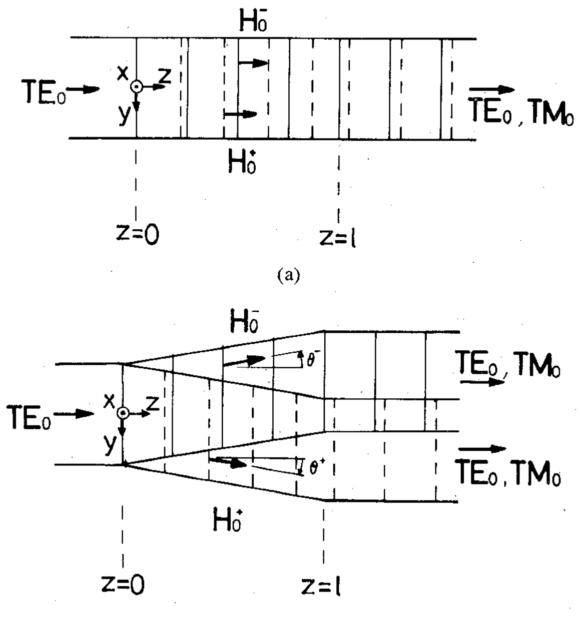

Fig. 2. Propagation path of optical beam waves for (a) linearly polarized hybrid modes H_0^+ and H_0^- ($\theta^+ = 0$, $\theta^- = 0$) and (b) circularly polarized hybrid modes H_0^+ and H_0^- ($\theta^+ \neq 0$, $\theta^- \neq 0$). The vertical solid and broken lines denote the equiphase planes. The arrows denote the direction of power flow.

It is assumed that the variation of $G(y)$ is small enough to be neglected compared with that of $e_y(x)$.

Let us assume that the TE_0 mode of unit power is incident from $z < 0$ in section I. Then, there propagate the H_0^+ and H_0^- modes in section II and the TE_0 and TM_0 modes in section III, where the effects produced by the discontinuity of the waveguide at $z = 0$ and $z = l$, such as the generation of reflected waves and higher order modes, are ignored. Application of the boundary conditions at $z = 0$ and $z = l$ gives the TE–TM mode conversion efficiency [5], [6]. Here, it is noted that the power of the H_0^+ and H_0^- modes flows in the direction of θ^+ and θ^- with respect to the wave-normal, respectively, as shown in Fig. 2. Based on the assumption that except for the neighborhood of the cutoff, most of energy of the guided modes is confined in the film and little in the top layer and substrate, we obtain the conversion efficiency:

$$\gamma_{ME} = (1 - \delta \cos \Delta\beta l)/2 \quad (3)$$

$$\gamma_{EE} = (1 + \delta \cos \Delta\beta l)/2 \quad (4)$$

$$\delta = \exp\left[-\left\{\frac{(\tan\theta^+ - \tan\theta^-) l}{\sqrt{2} W}\right\}^2\right] \quad (5)$$

$$\Delta\beta = \beta^+ - \beta^- \quad (6)$$

where γ_{ME} and γ_{EE} are the TE → TM and TE → TE mode conversion efficiencies (of power), respectively. β^+ and β^- are phase constants of the H_0^+ and H_0^- modes, respectively. The δ represents the overlap integral of the two hybrid modes.

As an example, we consider a waveguide structure proposed for nonreciprocal devices [4] with some modification: The materials of the top layer, film, and substrate are a nonmagnetized YIG ($n_t = 2.228$), As_2S_3 ($n_f = 2.46$), and $LiNbO_3$ ($n_o = 2.228$, $n_e = 2.151$), respectively, where[1] $\zeta = 0°$ for $z < 0$ and $l < z$ and $\zeta = 1°$

[1]This situation may approximately correspond to a thin-film optical waveguide, in which an external electric field is applied to the substrate (of electrooptic medium) only in the region $0 < z < l$ and is not in the regions $z < 0$ and $l < z$.

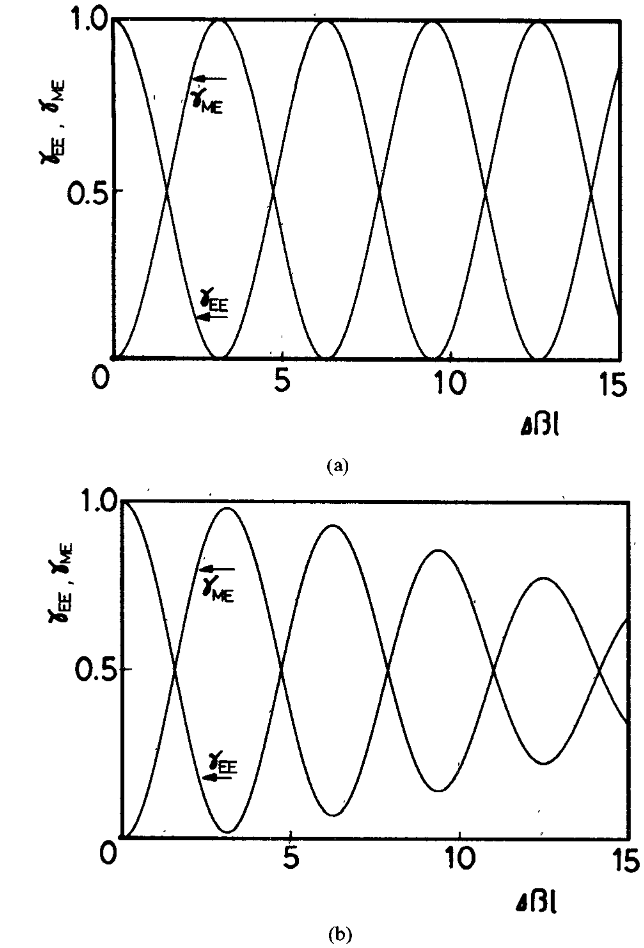

Fig. 3. TE–TM mode conversion efficiency for (a) linearly polarized hybrid modes and (b) circularly polarized hybrid modes.

for $0 < z < l$. The wavelength in vacuum is $\lambda_0 = 1.15\ \mu$m, and the film thickness[2] is $d_0 = .2262\lambda_0$. The hybrid modes are circularly polarized because we have anisotropic polar configuration in this case. Then, the hybrid mode beam waves propagate as shown in Fig. 2(b) ($\tan\theta^+ = +0.001475$, $\tan\theta^- = -0.001643$). The conversion efficiency is shown in Fig. 3(b), where the beam width is $W = 50\ \mu$m.

In the case of pure imaginary[3] ϵ_{23} (i.e., gyrotropic polar case), the hybrid modes have linear polarizations perpendicular to each other. Then, $\theta^+ = 0$ and $\theta^- \doteq 0$. The propagation path of the beam waves and the conversion efficiency are shown in Fig. 2(a) and Fig. 3(a), respectively. Complete TE–TM mode conversion is available at $\Delta\beta l = (2n-1)\pi$, $n = 1, 2, \cdots$.

III. Conclusions

We have given TE–TM mode conversion efficiency when a Gaussian beam wave propagates in thin-film optical waveguide with uniaxial anisotropic substrates. It should be noted that the direction of power flow of hybrid modes depends on the polarization when the TE–TM mode conversion is described as coupling between the two hybrid modes. For efficient TE–TM mode conversion, it is desirable that the uniaxial anisotropic substrates have a configuration sufficient to produce the linearly polarized hybrid modes, i.e., anisotropic longitudinal and gyrotropic polar configurations. The property of oblique power flow of the hybrid modes may be useful for thin-film waveguide-type optical deflectors and optical power dividers.

[2]In this example the confinement of the optical field in the guiding region may not be very strong. The conversion efficiencies of Fig. 3 are approximate ones.

[3]The ϵ_{ij}'s of dielectric tensor are real for the rotation of the optic axis of dielectric crystals, but here imaginary ϵ_{ij} is introduced to stand for gyrotropic media. For the direction of external magnetic field in the case of gyrotropic media, see [1, fig. 2].

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Two-Layer Dielectric Microstrip Line Structure: SiO_2 on Si and GaAs on Si: Modeling and Measurement

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Abstract—**Further development is reported of the modeling of the two-layer dielectric microstrip line structure by computing the scattering parameter S_{21} derived from the model and comparing the computed value with the measured value over the frequency range from 90 MHz to 18 GHz. The sensitivity of the phase of S_{21} and the magnitude of the characteristic impedance to various parameters of the equivalent circuit is**

Manuscript received June 25, 1987; revised October 30, 1987.
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